Correspondence

Parametric Amplification*

According to Manley-Rowe[1] power relations the difference of pump and signal frequencies yields the so-called negative resistance parametric amplifier. In this case, infinite gain is possible in contrast to the sum frequency amplifier or up-converter which has a finite gain determined by the pump and signal frequencies.

In order to realize parametric amplification, the circuit in question must contain a time varying or nonlinear reactive element which is usually accompanied by a nonlinear resistance such as the function of a semiconductor diode. Under this condition, the harmonic content of the resultant wave is very high, due to the existence of both amplitude and frequency modulation. Harmonic generators using this idea have been developed by Leenov and Uhlir.[2] Another condition which results in combined amplitude and frequency modulation is the nondissipative LC circuit with a time varying capacitor. Such a circuit involves the solution of the Mathieu's[3] equation given in (1)

$$q'' + \delta(1 + \cos W_s t)q = 0 \qquad (1)$$

where

$$\delta = \frac{(\text{natural frequency})^2}{(\text{signal frequency})^2}.$$

The sideband expressions of an amplitude modulation in presence of frequency modulation are given by (2) and (3).[4]

$$I_{F-f} = 0.5KI_M(B_0 + B_2)\cos(\Omega - w)t - B_1 I_M \cos(\Omega - w)t \qquad (2)$$

$$I_{F+f} = -0.5KI_M(B_0 + B_2)\cos(\Omega + w)t - B_1 I_M \cos(\Omega - w)t \qquad (3)$$

where

Ω = carrier or pump frequency
w = signal frequency
$B_0 = J_0(\beta)$
$B_1 = J_1(\beta)$
$B_2 = J_2(\beta)$

Eqs. (2) and (3) show a very important fact. The amplitude of the lower sideband is the difference of two factors as given in (4).

$$\{0.5K[B_1 + B_2] - B_1\}I_M = (c - d)I_M \qquad (4)$$

whereas the upper sideband is the sum of two negative factors. Under this condition, the magnitude of the lower sideband can be a very small percentage of the upper sideband. Therefore, one can talk of a negative type of parametric amplification if the product of parametric amplification gain and the amplitude of the lower sideband exceeds the amplitude of the upper sideband. Also, under very small lower sideband amplitudes, the signal-to-noise ratio may be a predominating factor in determining the quality of the amplifier. An important conclusion which can be drawn from the above explanation would be that up-converters may be superior to negative type of amplifiers in gain and noise considerations if the value of $(c-d)I_M$ goes to very low levels.

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[1] J. M. Manley and H. E. Rowe, "Some general properties of nonlinear elements, Part 1: General energy relations," PROC. IRE, vol. 44, pp. 904–913; July, 1956.

[2] D. Leenov and A. Uhlir, "Generation of harmonics and subharmonics at microwave frequencies with P-N junction diodes," PROC. IRE, vol. 47, pp. 1724–1729; October, 1959.

[3] J. R. Carson, "Notes on the theory of modulation," PROC. IRE, vol. 10, pp. 57–64; February, 1922.

[4] A. Hund, "Frequency Modulation," McGraw-Hill Book Co., Inc., New York, N. Y., p. 63, 1942.

A Microwave Technique for the Measurement of the Dielectric Properties of Soils*

During a recent investigation of antennas mounted flush with the earth, it was necessary to measure the dielectric properties of soil. Many different techniques for the measurement of dielectric constants and loss tangent have been developed.[1-2] These techniques generally fall into two categories: those which utilize transmission through a sample and those which use the reflection from the sample. The particular technique employed is modified to suit the range of parameters being measured and the physical characterisitics of the sample. In general, a measurement that provides an accurate determination of dielectric constant is relatively insensitive to variations in loss tangent and vice versa. The technique described here utilizes both types of measurements to find in a practical manner the dielectric constant and loss tangent of relatively moist loamy soil at microwave frequencies.

The attenuation of a wave propagating in the sample is measured by probing a soil-filled section of rectangular waveguide through a series of holes drilled along the centerline of the top face. A typical experimental setup is shown in Fig. 1 where the top face of the waveguide has been removed to show the sample of soil. The effect of the reflected wave in the soil may be neglected by making the sample sufficiently long. The magnitude of the field is then given by

$$E = E_0 e^{-(\alpha/k_0)k_0 x} \qquad (1)$$

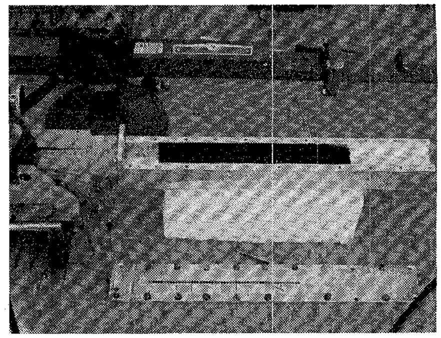

Fig. 1—Equipment for measurement of dielectric constant and loss tangent of soil.

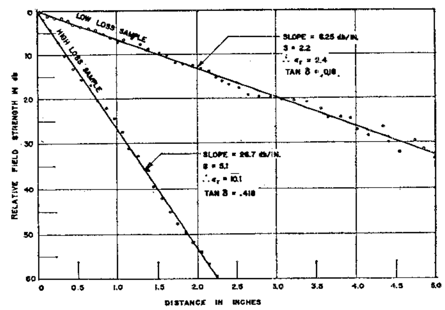

Fig. 2—Attenuation in waveguide filled with two different samples of soil.

where α is attenuation constant, k_0 is the free space propagation constant (*i.e.*, $k_0 = 2\pi/\lambda_0$ where λ_0 is free space wavelength), x is the distance measured along the waveguide, and E_0 is the magnitude of the field at the front edge of the sample. Thus, if the magnitude of the field in db is plotted against distance along the guide, the result is a straight line whose slope is proportional to α/k_0. Fig. 2 shows examples of probe data taken with two different samples of soil. It will be observed that there are slight variations from the straight line curve which are due largely to nonuniform density of the soil sample; however, with a sufficient number of points, the slope of the line is quite accurately determined. No essential variation from the straight line was observed with a homogeneous material such as soap.

The propagation constant (γ) for the dominant mode in the soil filled portion of the rectangular waveguide may be written (assuming $e^{j\omega t}$ time dependence) as

$$\gamma = \alpha + j\beta = k_0\left[\left(\frac{\lambda_0}{\lambda_c}\right)^2 - \epsilon_r + j\epsilon_r \tan\delta\right]^{1/2} \qquad (2)$$

where ϵ_r is the relative dielectric constant of the soil, $\tan\delta$ is the loss tangent of the soil, and λ_c is equal to twice the waveguide width. Solving for the loss tangent gives

$$\tan\delta = \frac{2}{\epsilon_r}\left(\frac{\alpha}{k_0}\right)\left[\left(\frac{\alpha}{k_0}\right)^2 + \epsilon_r - \left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{1/2}. \qquad (3)$$

* Received December 14, 1962; revised manuscript received January 2, 1963.

[1] A. R. Von Hippel, Ed., "Dielectric Materials and Applications," Technology Press of M.I.T. Cambridge Mass., and John Wiley and Sons, Inc., New York, N. Y., 1954.

[2] C. G. Montgomery, Ed., "Technique of Microwave Measurements," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Company, Inc., New York, N. Y., vol. 11; 1947.